Multi-period supply chain re-design in the electric motors industry
Model implementation and solution strategy

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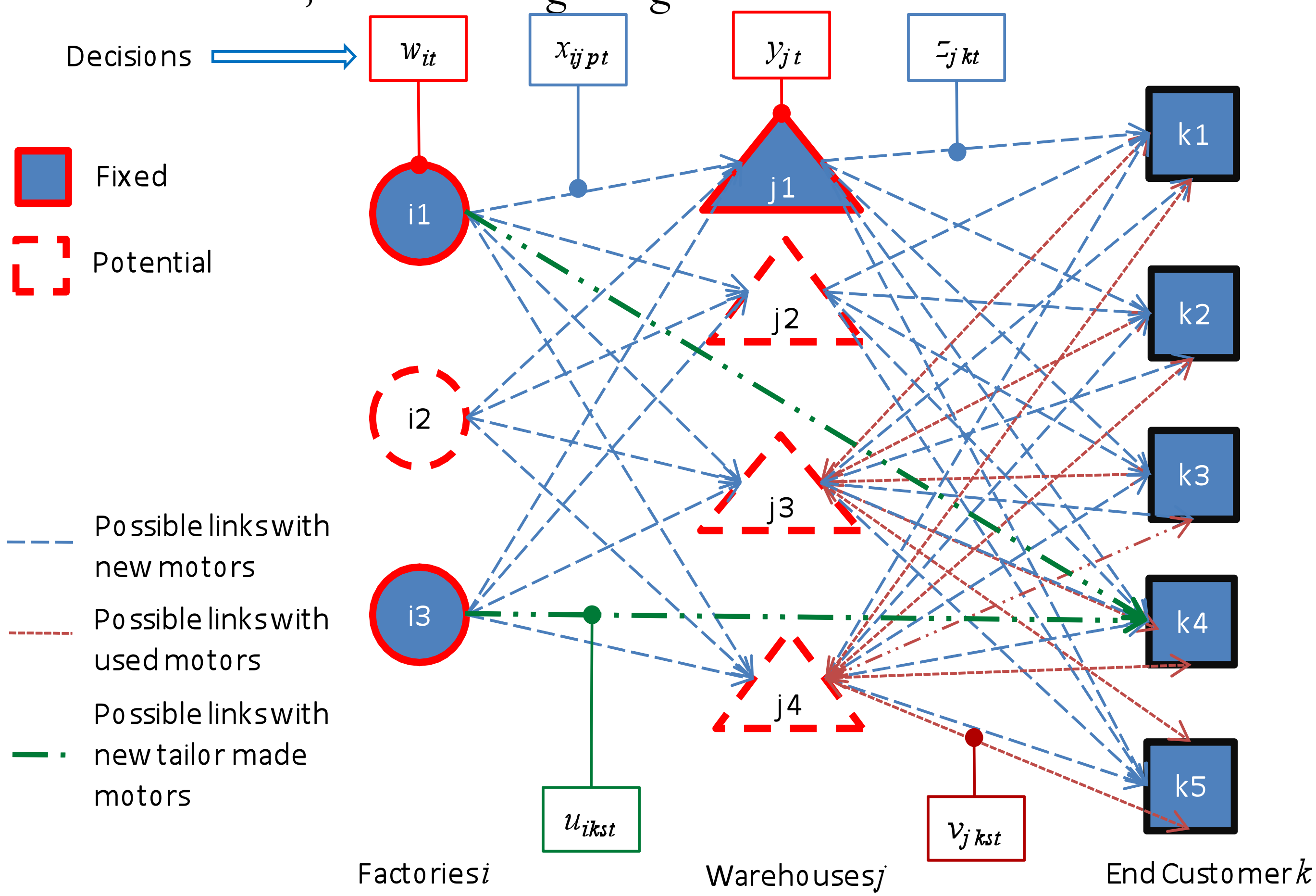
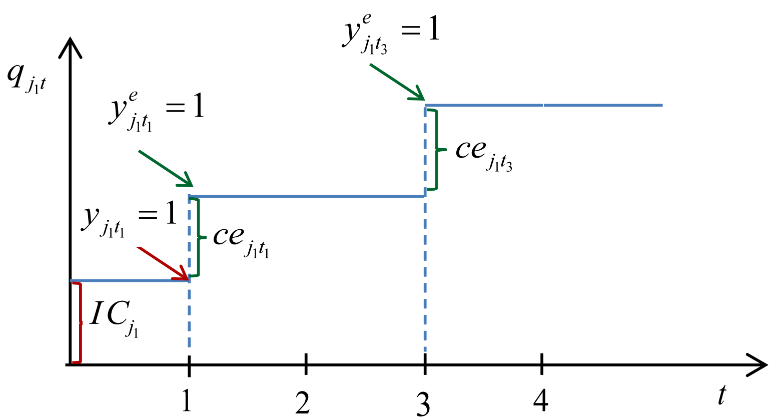
Supply chain superstructure

- Given a supply chain superstructure with Multi-product Factories i, Warehouses j and End Customers k, the following design decisions are considered:

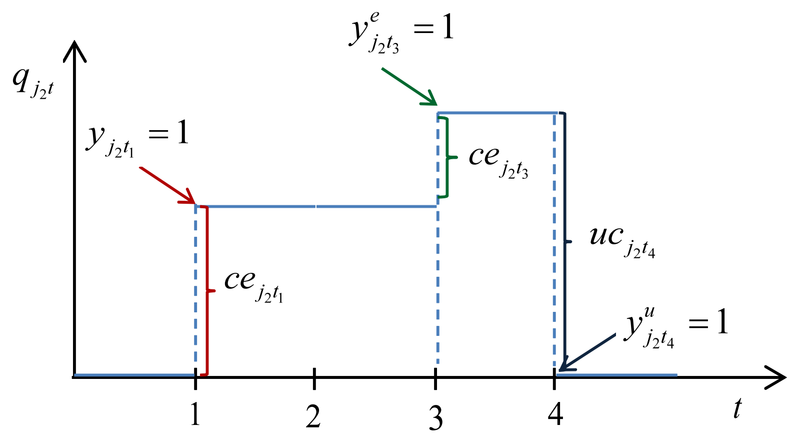


Capacity decisions

- Capacity profile decisions in the multi-period problem:



Warehouse j_1 which is fixed



Warehouse j_2 which is installed in period 1

Capacity decisions

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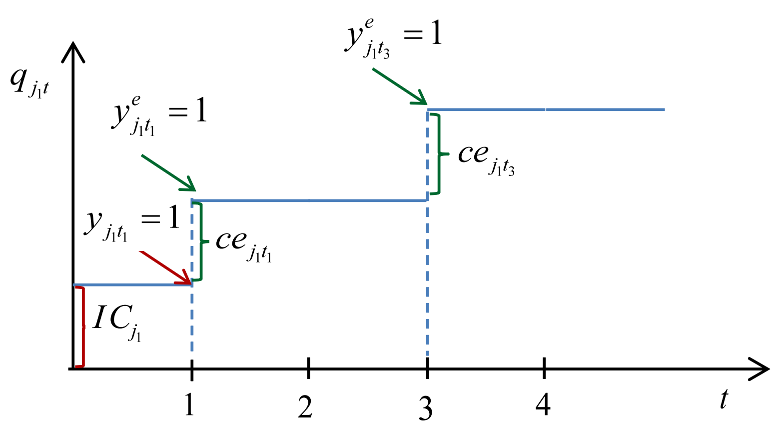

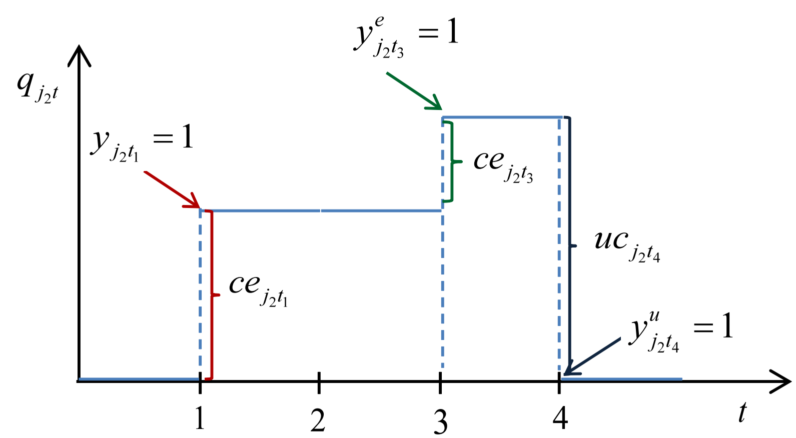

Warehouse j_1 which is fixed

Warehouse j_2 which is installed in period 1

Warehouse j_1 is already installed in the supply chain and the initial capacity is given by IC_{j_1}. It is expanded in the first period which is indicated by binary variable $y^e_{j_1 t_1} = 1$. Capacity expansion is given by $ce_{j_1 t_1}$. The same decision is made in period 3.

Capacity decisions

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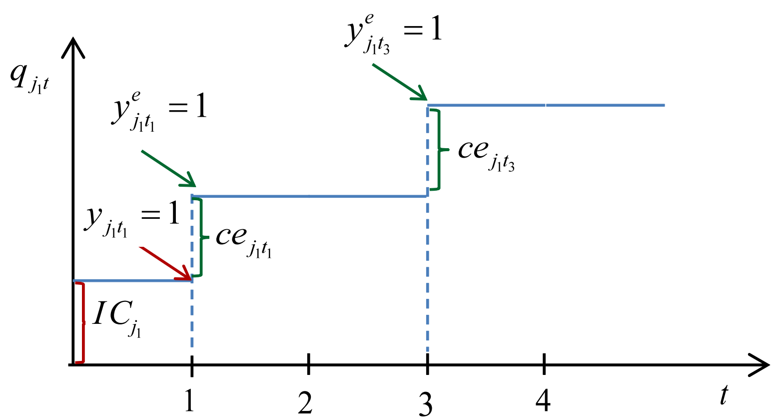

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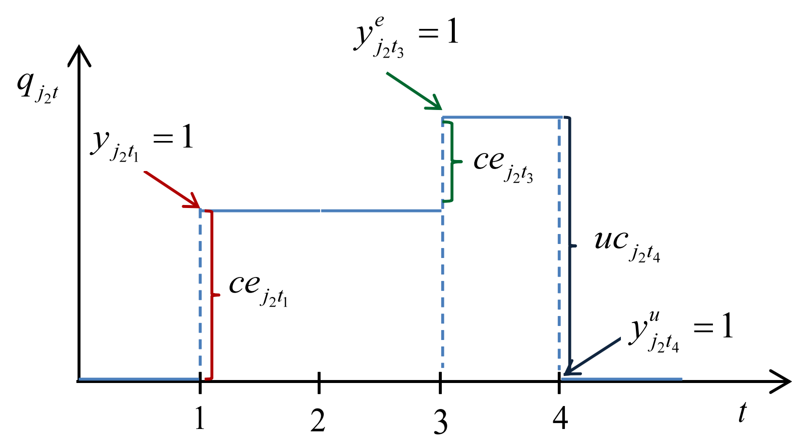


Warehouse j_2 which is installed in period 1

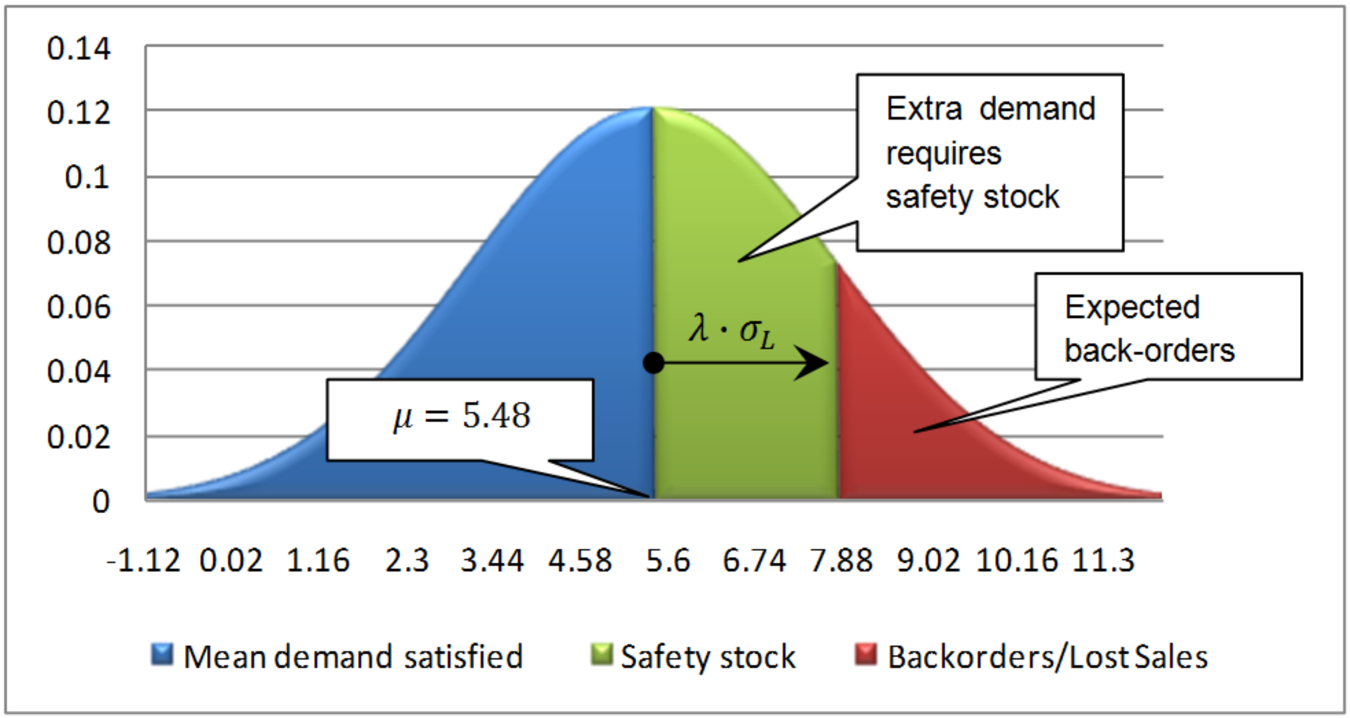
Warehouse j_2 is installed in period 1 which is indicated by $y_{j_2t_1} = 1$. This capacity expansion is given by $ce_{j_2t_1}$. This warehouse is expanded again in period 3. Therefore, total capacity in period 3 is given by $q_{j_2t_3} = ce_{j_2t_1} + ce_{j_2t_3}$. In period 4 this warehouse is eliminated from the supply chain, this is decided by $y^u_{j_2t_4} = 1$ and the capacity uninstalled is given by $uc_{j_2t_4}$.

Problem statement

- *Objective: Redesign an optimal supply chain for the electric motors industry minimizing costs and deciding where and when to place warehouses, which installed warehouses should be eliminated in each period, what are the stock capacities profiles and safety stocks required as well as how to connect the different echelons of the supply chain in order to satisfy uncertain demand of motors over a multi-period horizon planning.*
- Challenges:
  - **Multiple products** to deliver
  - **Uncertain Demand** with known probability distribution due to **motor failure rate** in End Customer Plants
  - Demand can be partially satisfied with **repaired motors**
  - **End customers** could have **storage** capacities
  - End customers expect different **service level** for their product (guaranteed time)
  - **Multi-period** scope
  - How to **efficiently implement** the original non-linear model

Demand, Safety stock & Back-orders

- Uncertainty modeled with Poisson distribution
  - Mean demand is given by the **average failure rate**
  - Standard deviation indicates demand (failure) **variability**



- How to face and diminish the EFFECT of demand uncertainty? → SAFETY STOCK (additional stock to face extra demand)
- Any demand exceeding target demand → Back-orders/lost sales

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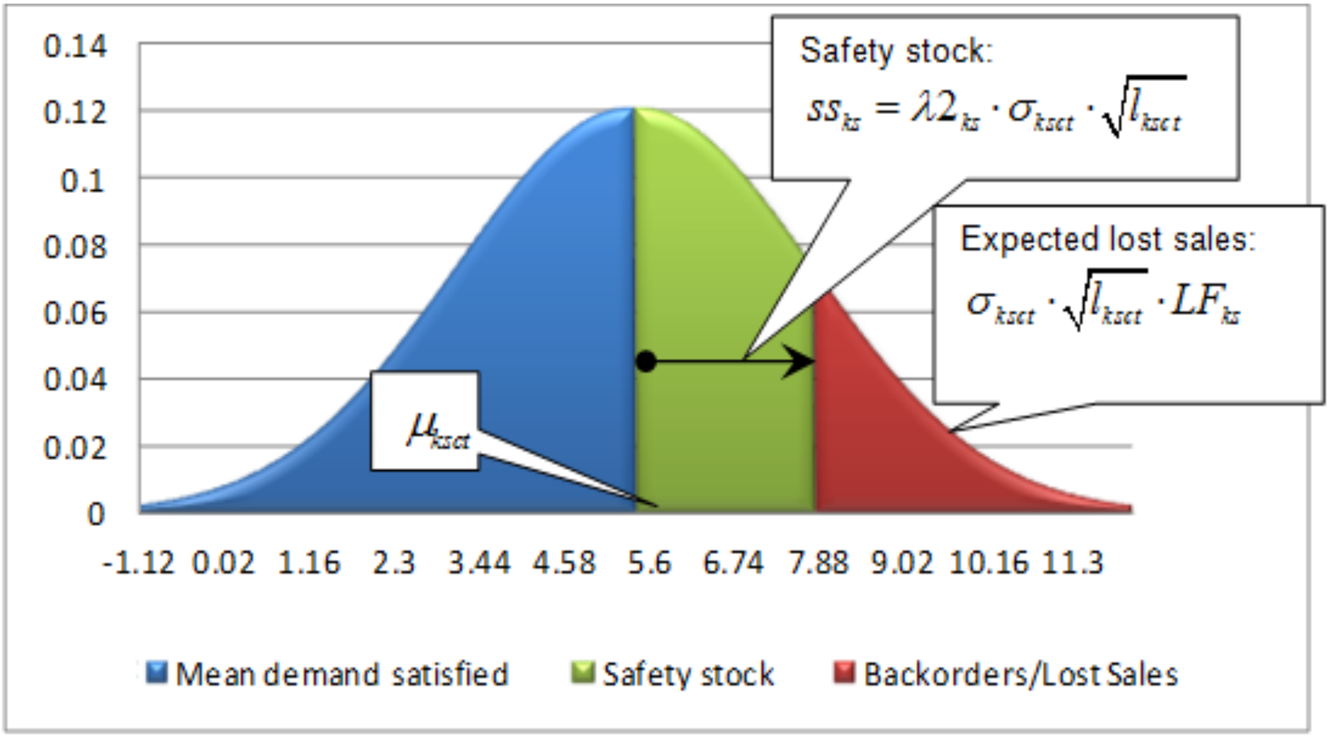

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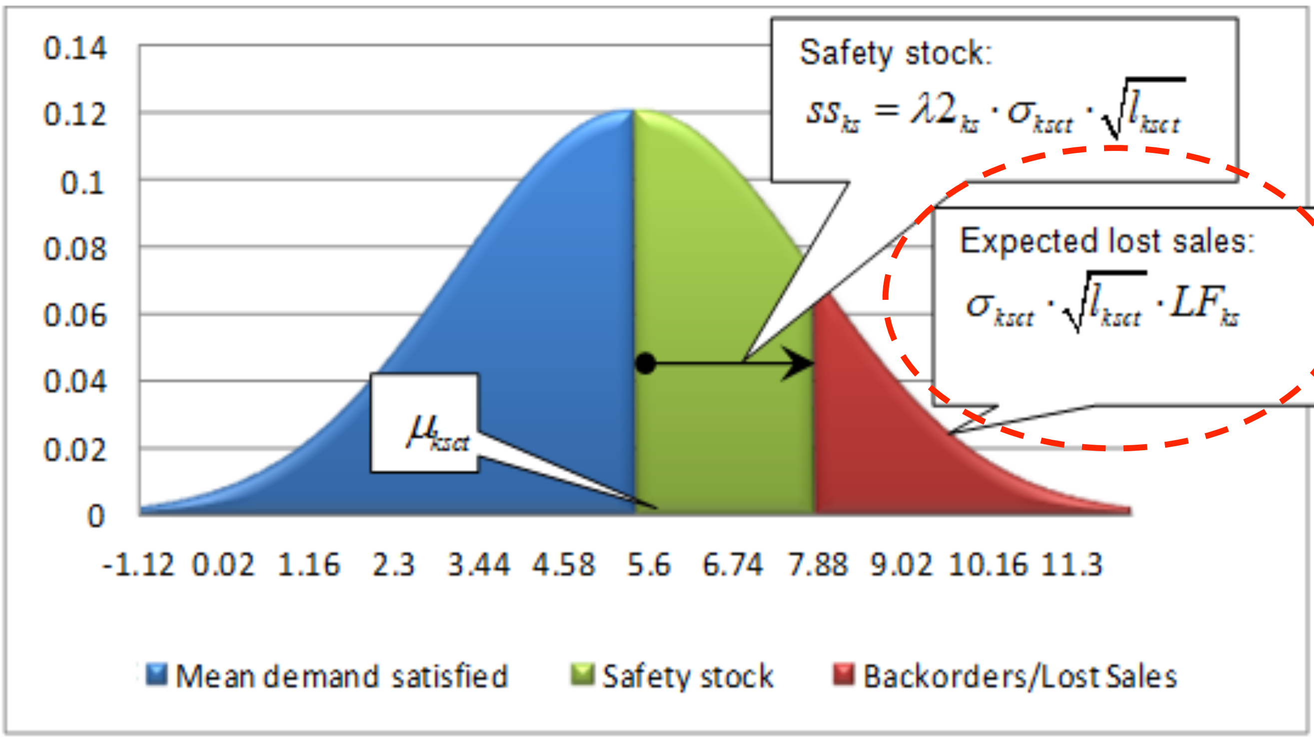

Adapted from Parker and Little (2006)

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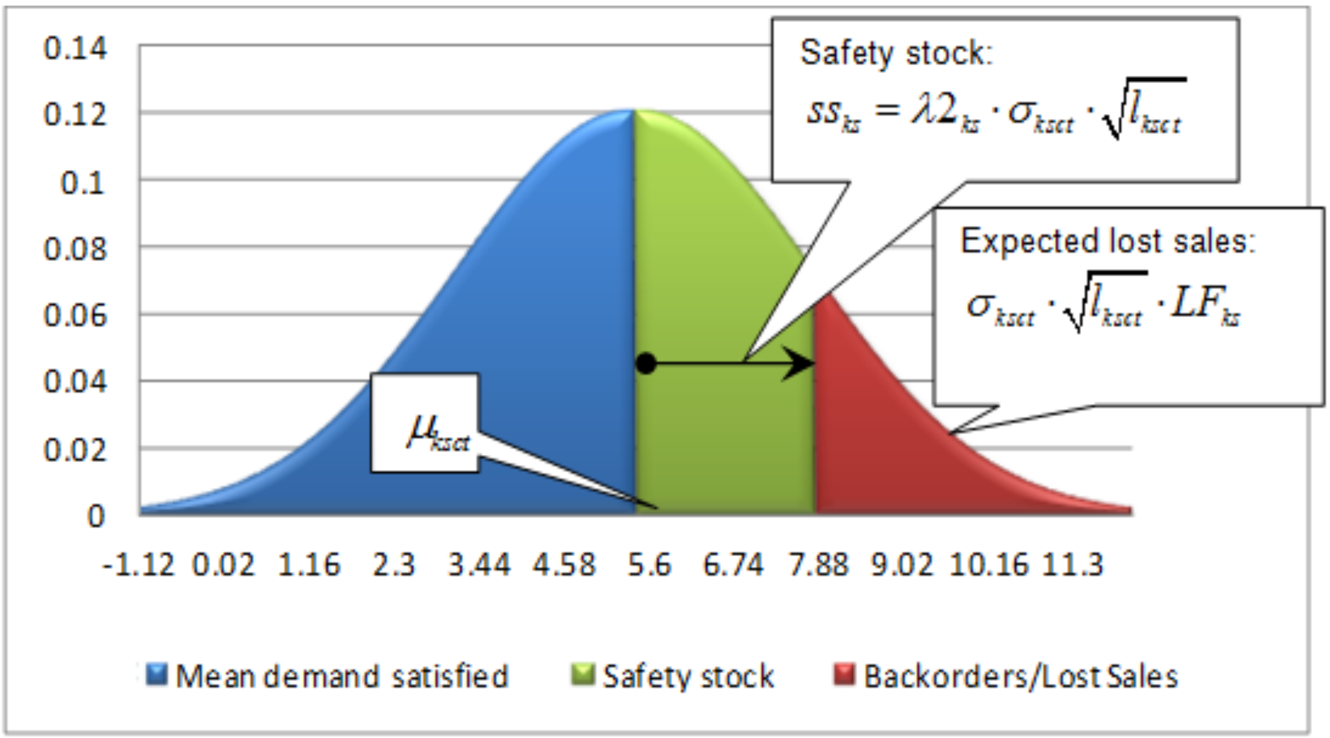
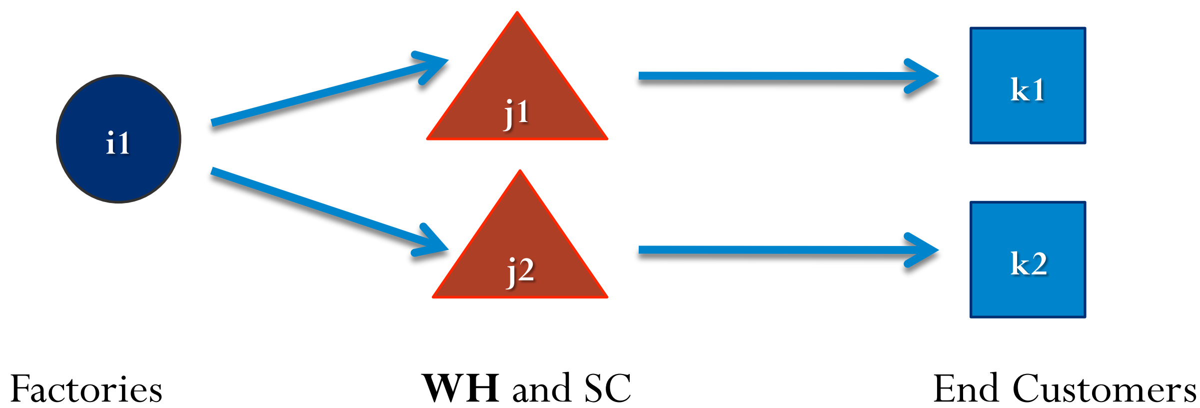


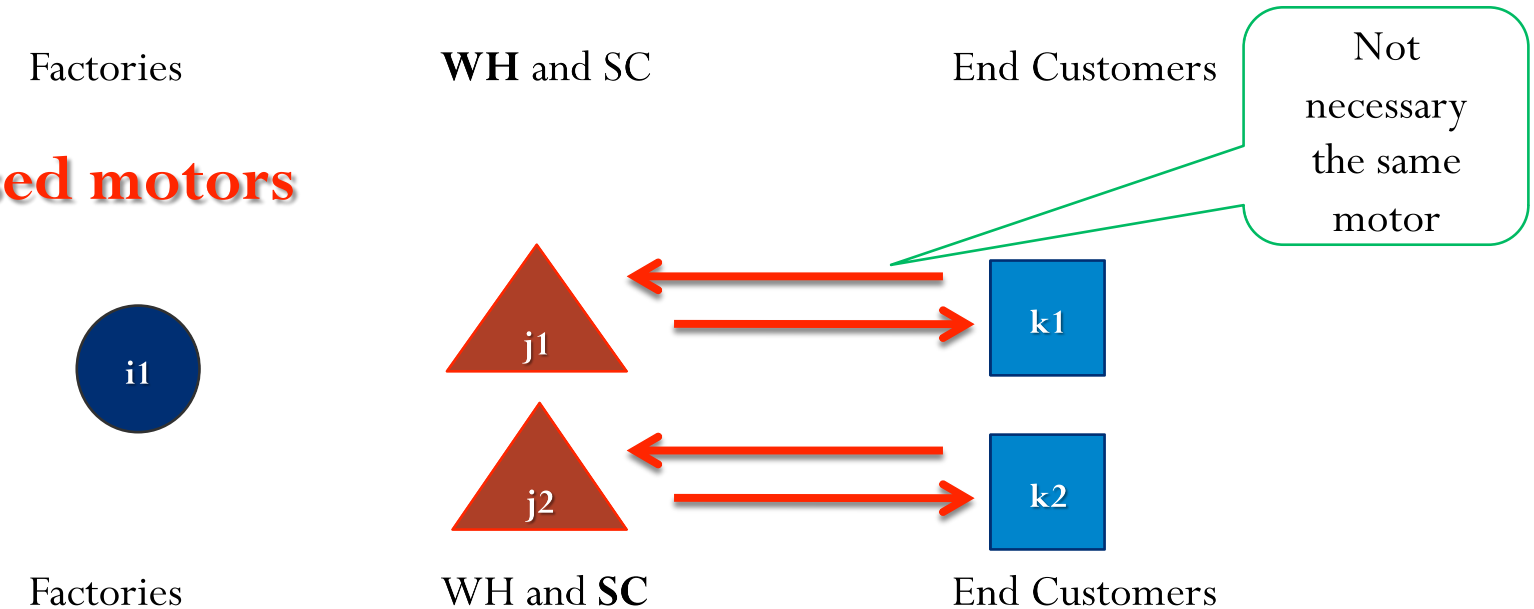
- **Cost of Lost sales:** $$CB_{ksct} = b1_{ks} \cdot \sigma_{ksct} \cdot \sqrt{l_{ksct}} \cdot 0.45 \cdot e^{\lambda 2_{ks}/-0.59} \cdot \chi \cdot \sum_j \frac{z_{jkt}}{t2_{jkp}}$$

Motors Route

- **New motors**



- **Used motors**



Objective Function & Constraints

- Costs in **the objective function:**
  - Installation/Expansion of Warehouses and Repair work-shops
  - Operating Fixed Costs of installed warehouses
  - Elimination Cost of installed Warehouses
  - Processing and repairing Costs
  - Inventory handling Costs of new motors and repaired motors
  - Transportation Costs
  - Safety stock Costs
  - Lost sales Costs due to Stock shortage
- Constraints:
  - Logic constraints to assure coherence when assigning links in the supply chain
  - Demand constraints to define how demand is satisfied: new and used motors,
  - Capacity constraints in factories and warehouses
  - Net lead time definitions in the different echelon of the supply chain using guaranteed service time approach

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Investment Costs

Objective Function & Constraints

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Operational Costs

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  - Elimination C...
  - Processing an...
  - Inventory han...
  - Transportatio...
  - Safety stock C...
  - Lost sales Cos...

Remarks:

- ❑ Original problem formulated as MINLP problem
- ❑ Bilinear terms
- ❑ Square root in safety stock and lost sales costs
- ❑ Original formulation relaxed as a linear model applying piece wise linearization of the square roots that appear in the objective function and applying exact linearization of bilinear equations

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Model implementation

- MINLP model is transformed into a MILP relaxation:
  - An **exact linear** transformation is applied for **BILINEAR** terms → New Variables and Constraints
  - An **approximated** linearization (Lower Bound) is applied for **SQUARE ROOT** terms

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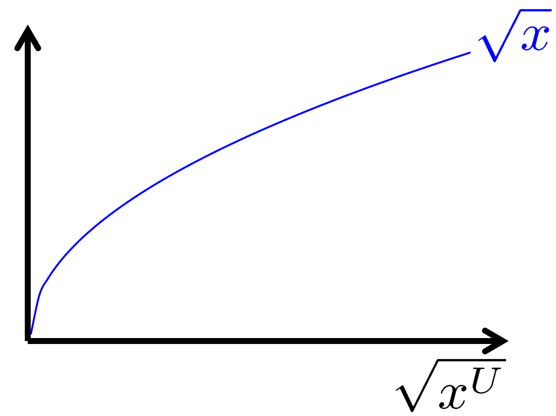
Sequential linearization

Piece-wise linear approximation

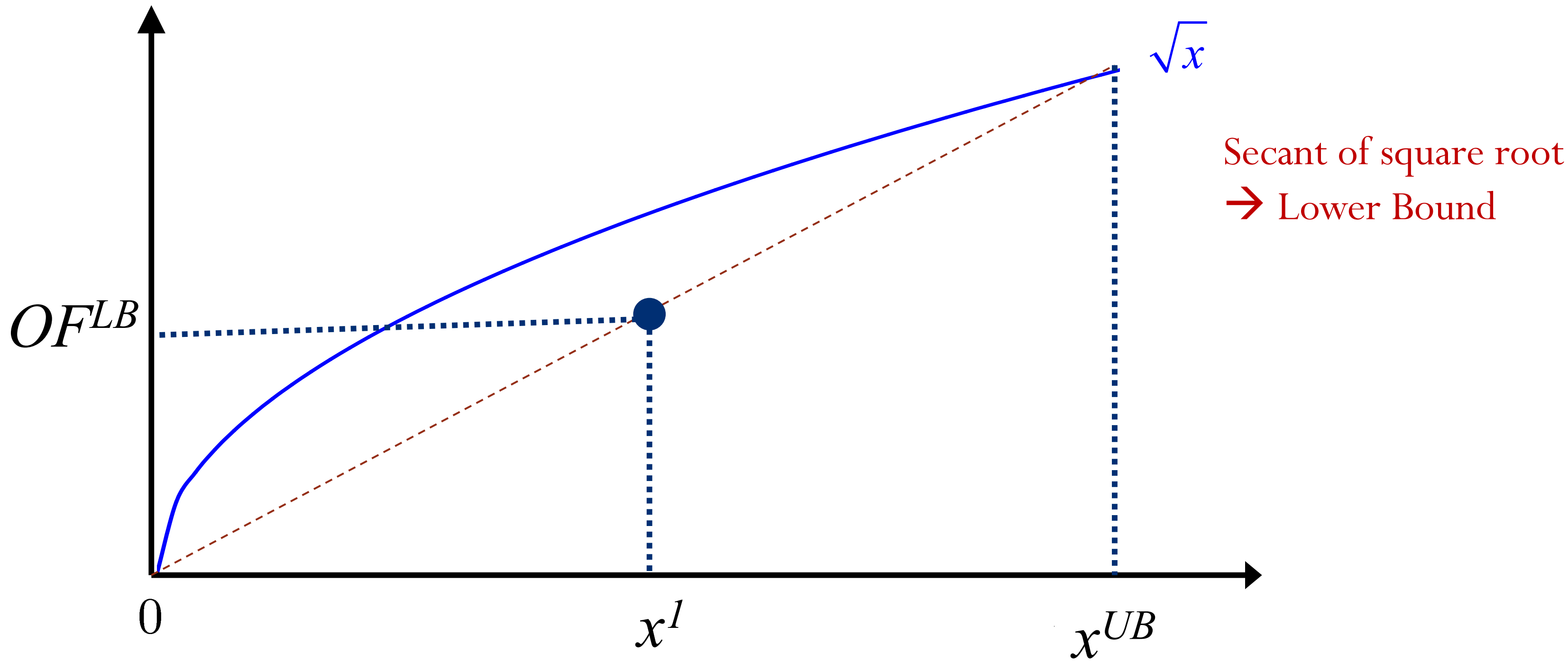
- Non linear terms: square root terms



- Approximate Univariate Square Root Terms
  - Piece-wise linear approximation (MILP)
  - Dynamic strategy for selecting the UB and LB of intervals
  - MILP provides a valid global lower bound of the original MINLP

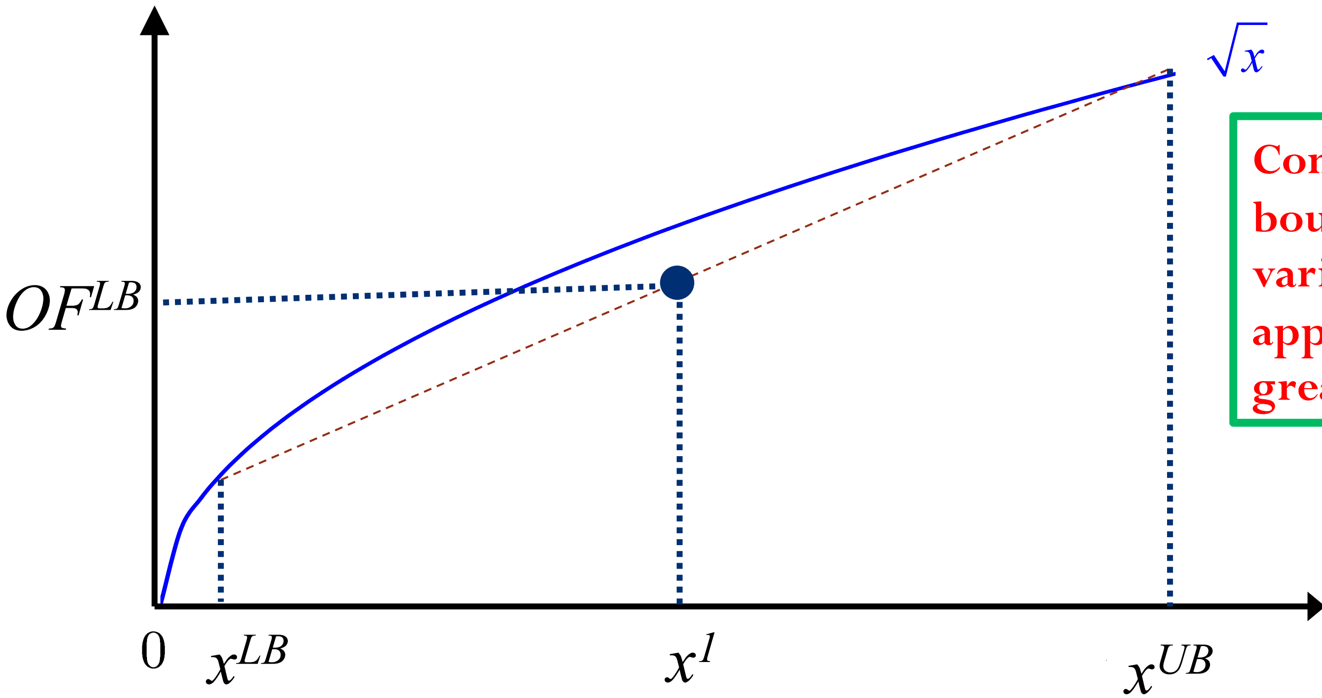
Piece-wise linear approximation

- Procedure: first step



Piece-wise linear approximation

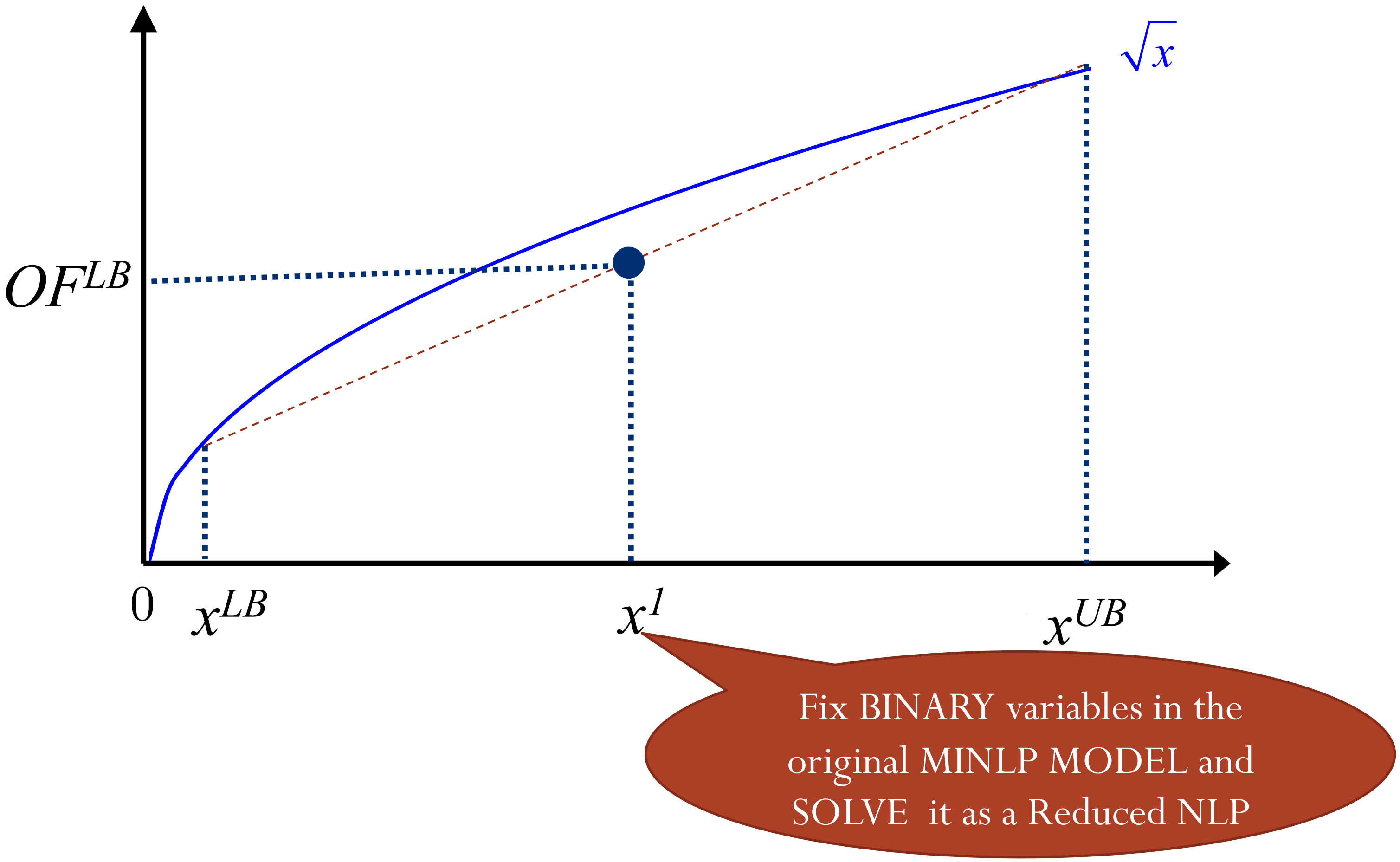
- Procedure: **improved** first step



Considering lower bound of the variable the approximation is greatly improved

Piece-wise linear approximation

- Procedure: second step



Piece-wise linear approximation

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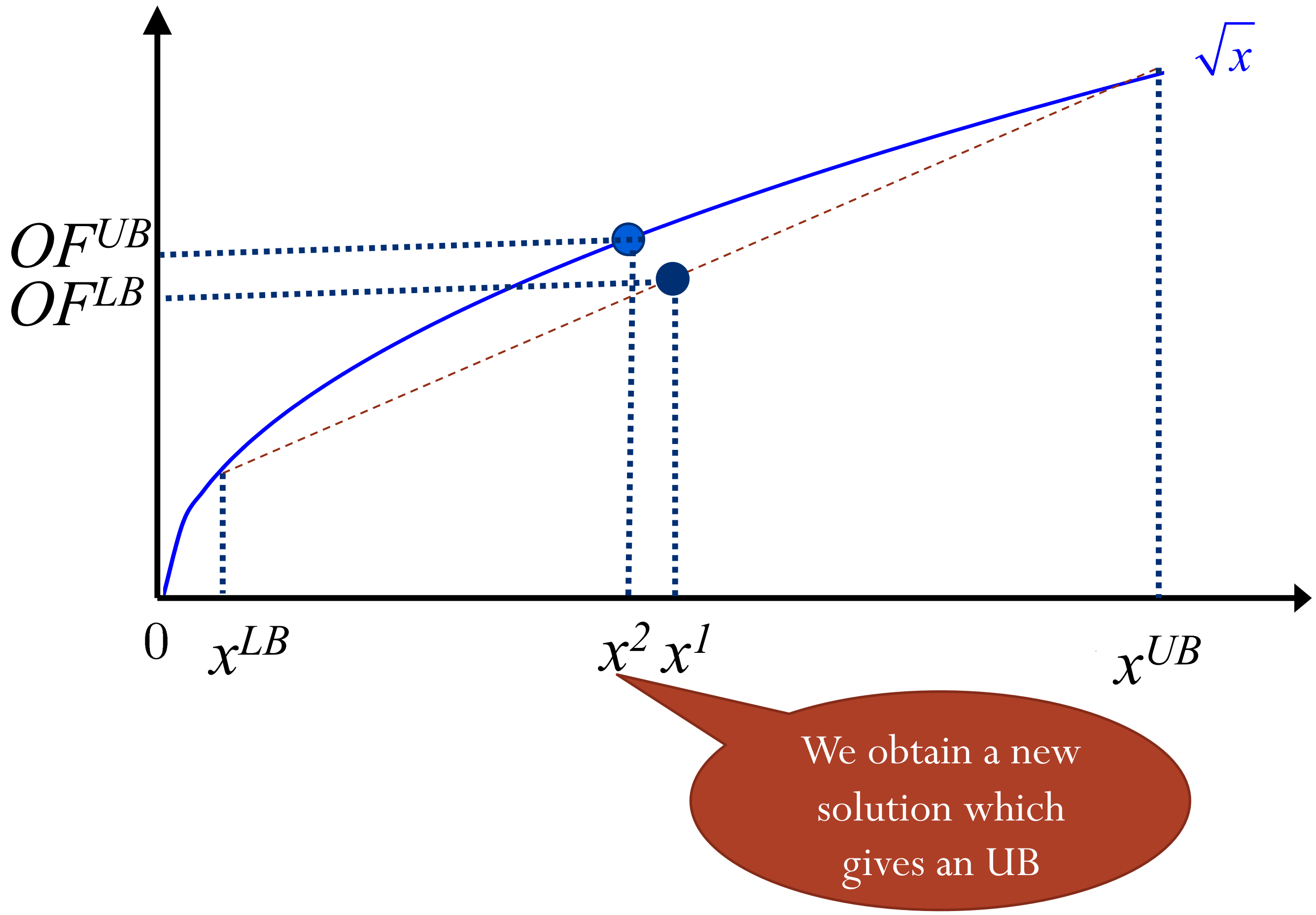
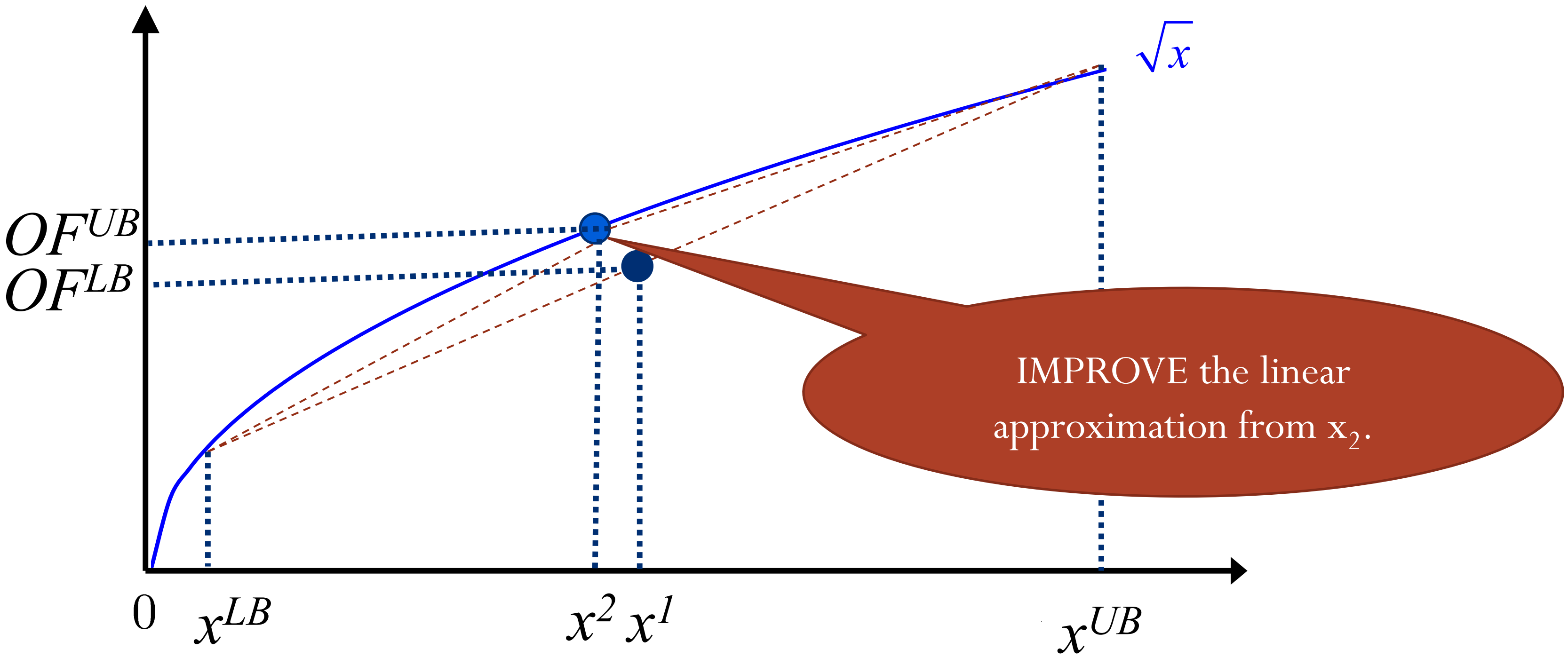


Piece-wise linear approximation

- Procedure: third step



Piece-wise linear approximation

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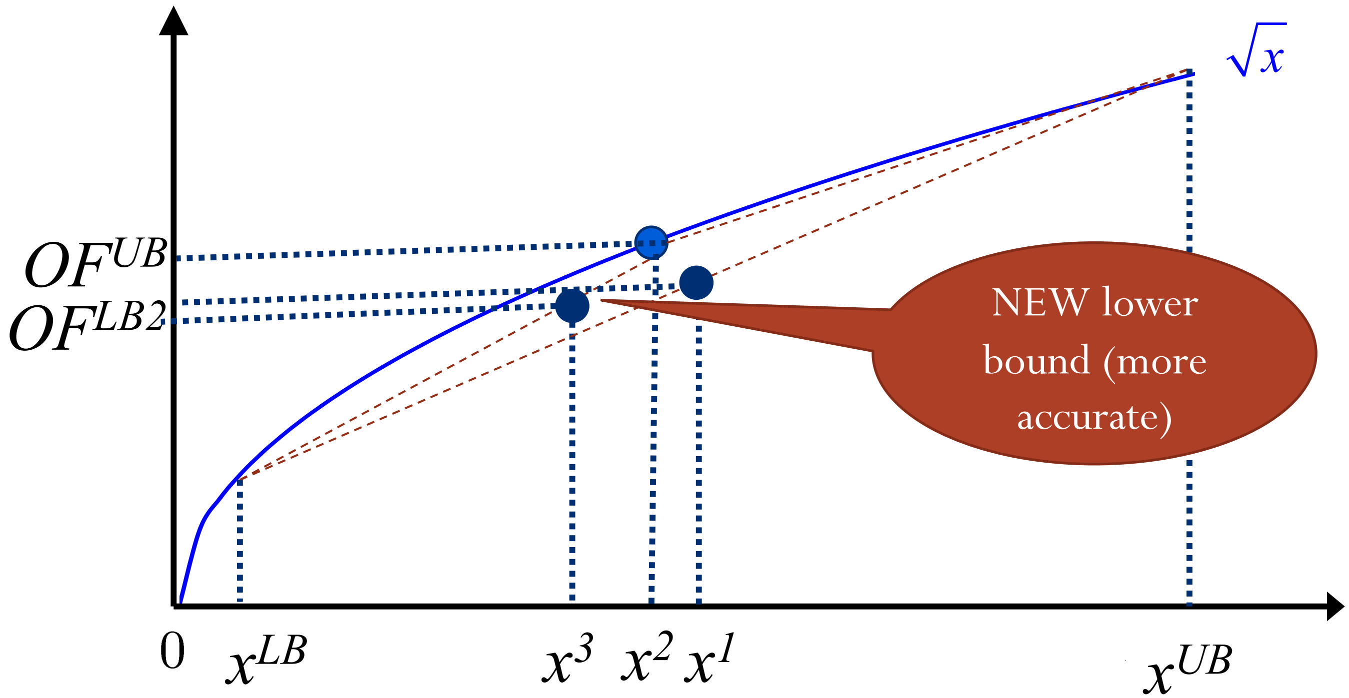
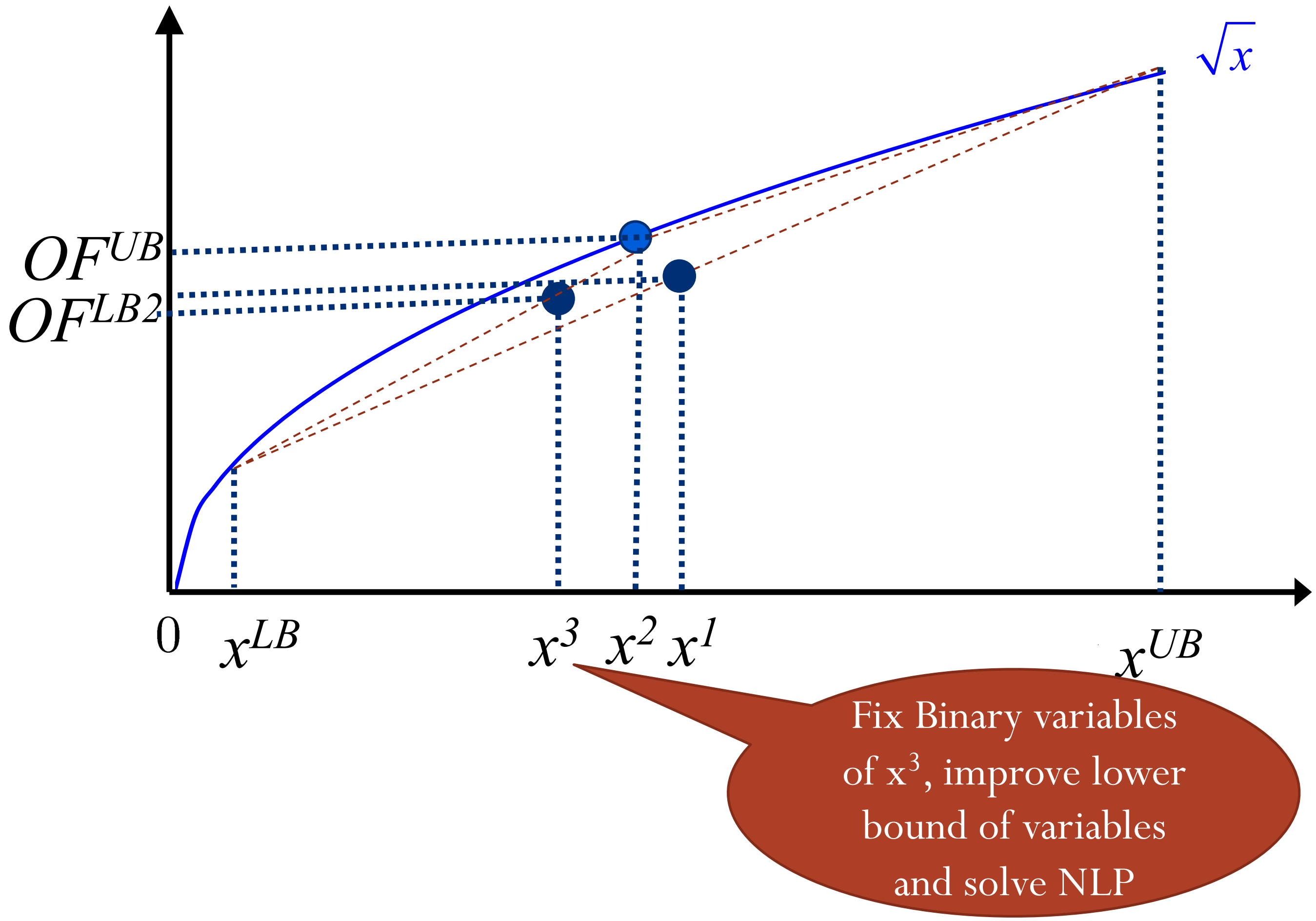


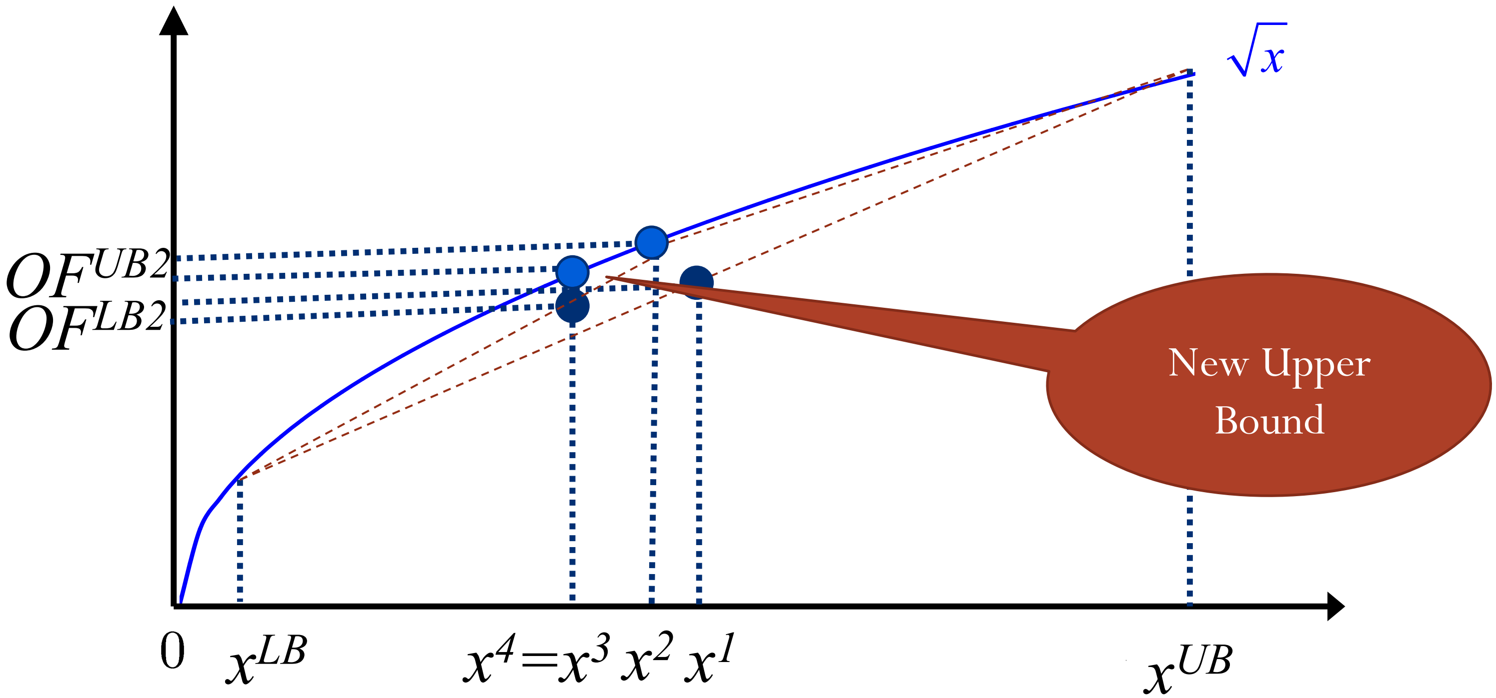
Piece-wise linear approximation

- Procedure: fourth step



Piece-wise linear approximation

- Procedure: fourth step



Example solution

- Problem size:

	Multiperiod Problem Size:	
i	factories	4
j	warehouses	10
k	customers	20
p	standard motors	5
s	special motors	11
t	periods	5
c	motors criticality levels	4

Procedure performance

- Models size and execution time

Iterations:	Equations	Positive Variables	Binary Variables	OF	Status	UB-LB gap (best values)	CPUs
MILP1 (LB)	108064	57193	4407	47640749	Optimal	6.90%	33.04
NLP1 (UB)	37274	22906	0	50925931.7	Locally optimal		17.91
MILP2 (LB)	114504	64988	6822	48053420.43	Integer Gap **0.35%**	2.82%	300.53
NLP2 (UB)	37274	22906	0	49406853.34	Locally optimal		57.46
MILP3 (LB)	116114	65793	7627	48290602.47	Integer Gap **1%**	2.31%	127.06
NLP3 (UB)	37274	22906	0	49570667.22	Locally infeasible		6.97

New Challenge

- The model size is increased in each iteration because new binary and continuous variables as well as constraints are introduced in each linearization step.
- A Lagrangean Decomposition Technique will be applied to solve the problem in reasonable execution time for large problems.

Thanks for your attention!

Questions?